Asymptotic Analysis

Primality Testing

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Definition

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Definition: Let $num \geq 2$ be an integer.

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Examples:

13 is prime

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13 is prime 12 is not prime

Definition: Let $num \geq 2$ be an integer, and let d and k be two divisors of num. We say that *k and d are complementary divisors* of num

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Examples:

4 and 25 are complementary divisors of 100

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Examples:

13 is prime 12 is not prime

Definition: Let $num \geq 2$ be an integer, and let d and k be two divisors of num. We say that *k and d are complementary divisors* of num, if $d \cdot k = num$

Examples:

4 and 25 are complementary divisors of 100

5 and 20 are complementary divisors of 100

Primality Testing

Version I: $\underbrace{1, 2, 3, \quad . \quad . \quad . \quad , num}$

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Version II:

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num = 100:

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num = 100: 1 2 4 5 10 20 25 50 100

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num = 100: 1 2 4 5 10 20 25 $\underset{\nwarrow \frac{100}{2}}{50}$ 100

Primality Testing

$$1,2,3, \quad . \quad . \quad . \quad , \frac{num}{2}, \quad . \quad . \quad . \quad , num$$

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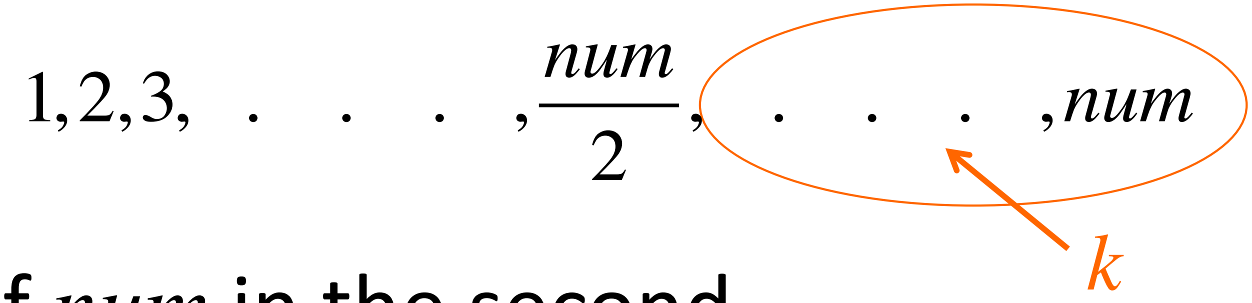
$$1,2,3, \quad . \quad . \quad . \quad , \frac{num}{2}, \underbrace{\quad . \quad . \quad . \quad , num}_{k}$$

Primality Testing



$$1,2,3,\quad .\quad .\quad .\quad ,\frac{num}{2},\quad .\quad .\quad .\quad ,num$$

Let k be a divisor of num in the second half of the range.

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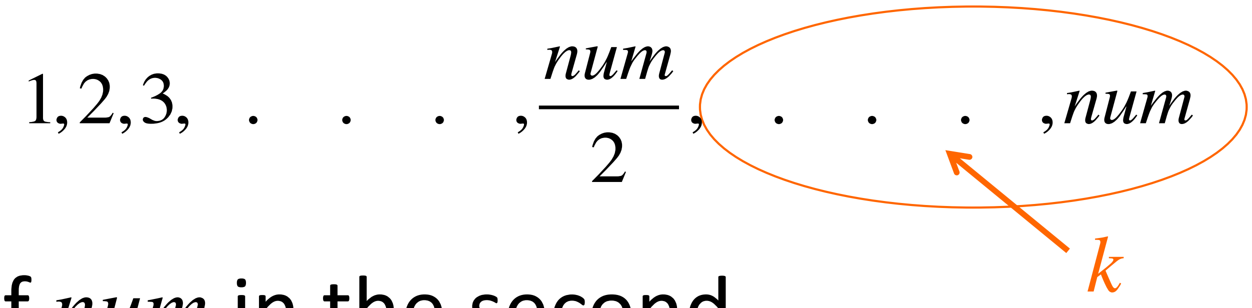

$$1, 2, 3, \quad . \quad . \quad . \quad , \frac{num}{2}, \quad . \quad . \quad . \quad , num$$

Let k be a divisor of num in the second half of the range. That is, $k > \frac{num}{2}$

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$$1,2,3,\ .\quad .\quad .\ ,\frac{num}{2},\underbrace{\ .\quad .\quad .\ ,num}_{k}$$

Let k be a divisor of num in the second half of the range. That is, $k > \frac{num}{2}$

Let d be k’s complementary divisor

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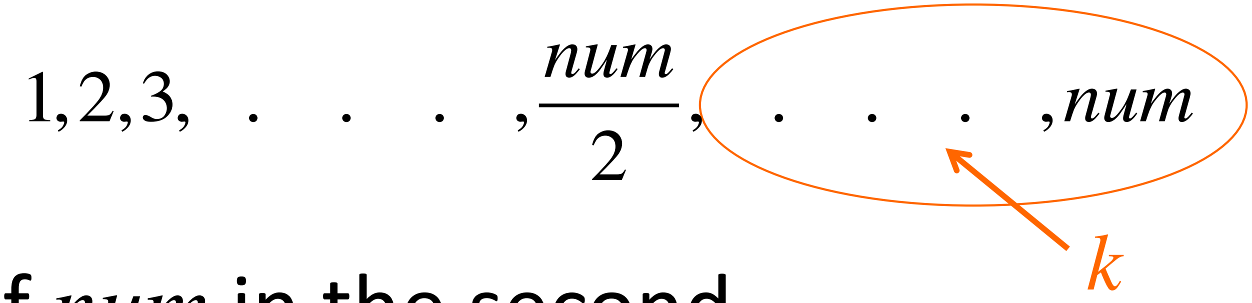

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Let k be a divisor of num in the second half of the range. That is, $k > \frac{num}{2}$

Let d be k's complementary divisor, therefore $d = \frac{num}{k}$

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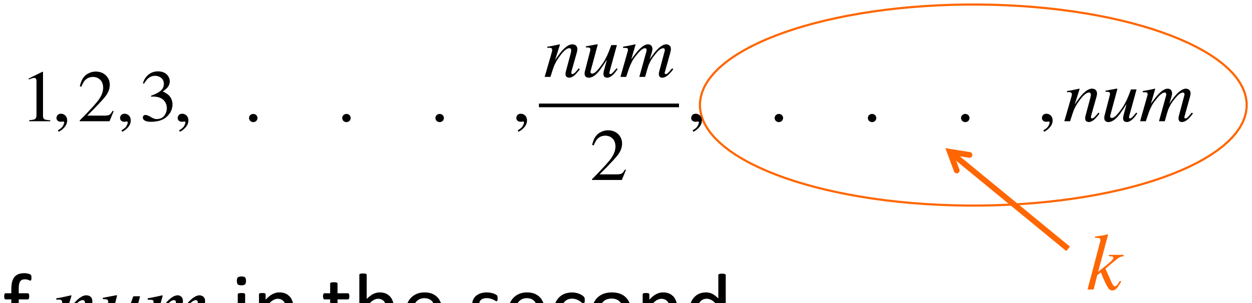

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Let d be k's complementary divisor, therefore $d = \frac{num}{k}$

We have:

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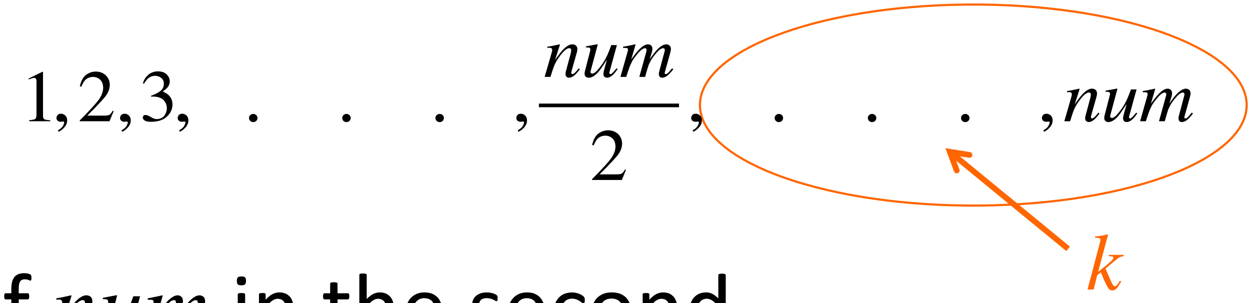

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Let d be k's complementary divisor, therefore $\boxed{d = \frac{num}{k}}$

We have: $d = \dfrac{num}{k}$

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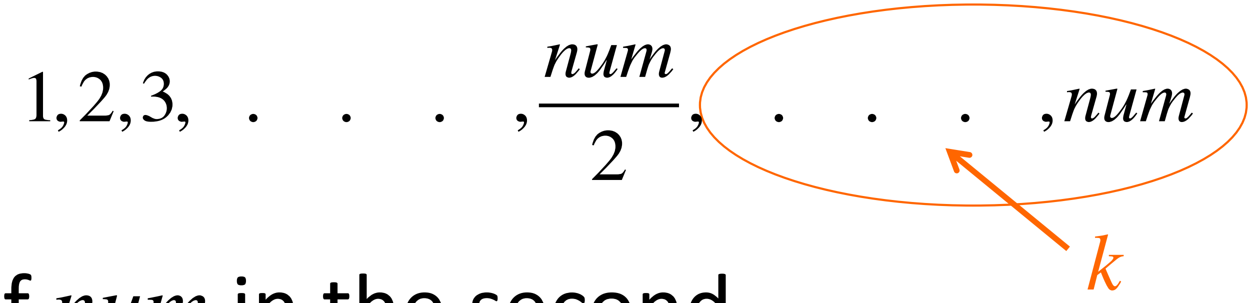

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Let d be k's complementary divisor, therefore $d = \frac{num}{k}$

We have: $d = \dfrac{num}{k} \ ? \ \dfrac{num}{\left(\frac{num}{2}\right)}$

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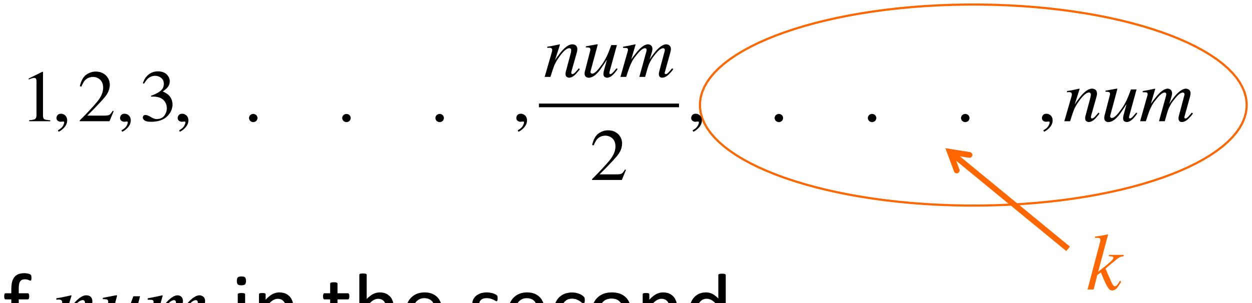

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We have: $d = \dfrac{num}{k} < \dfrac{num}{\left(\frac{num}{2}\right)}$

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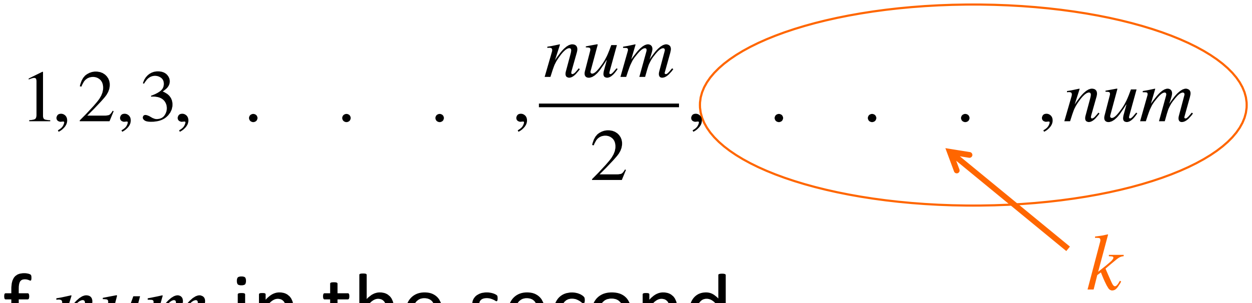

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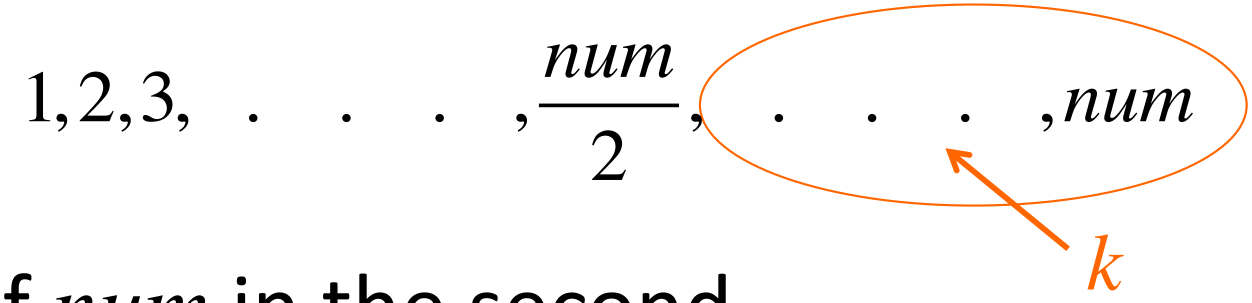

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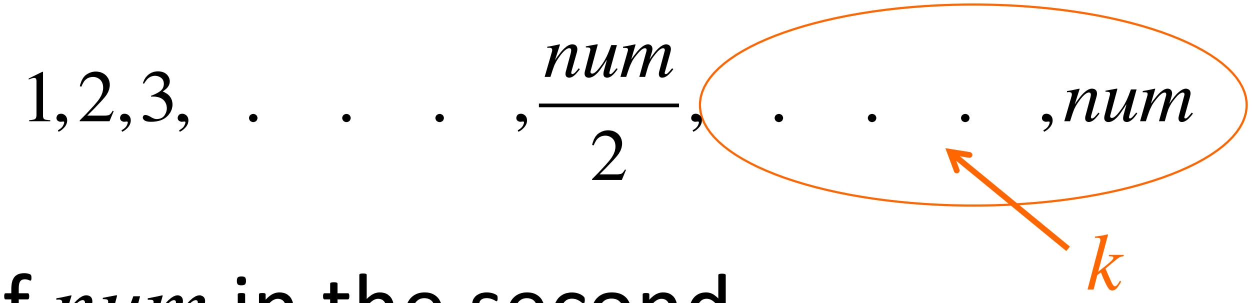

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Therefore $d<2$.

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Therefore $d<2$. Since d is a divisor

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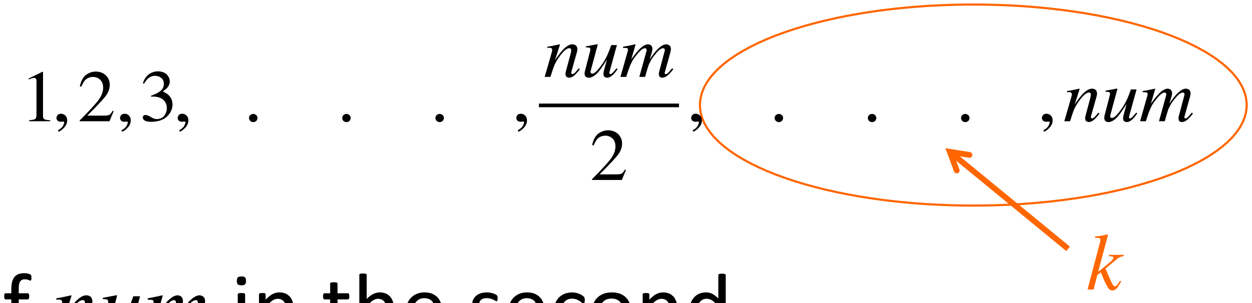

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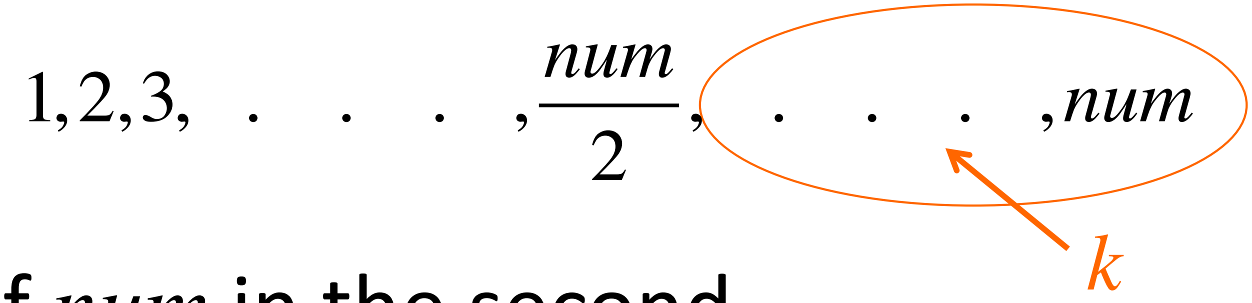

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Therefore $d<2$. Since d is a divisor, we get that $d=1$.

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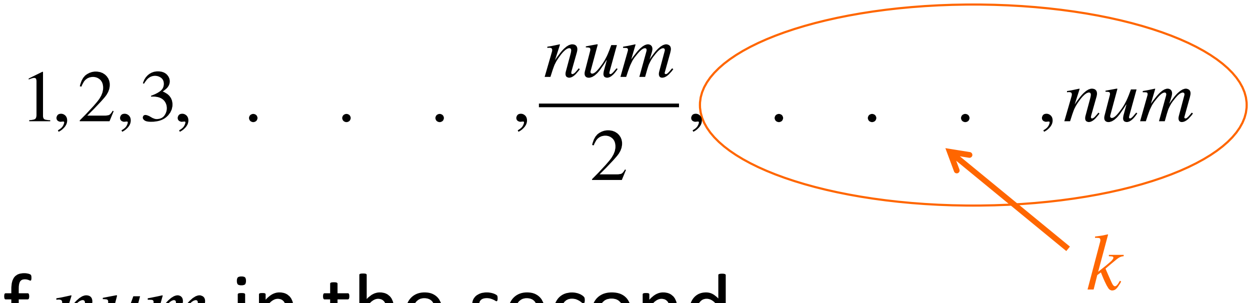

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So: $\frac{num}{k} = 1$

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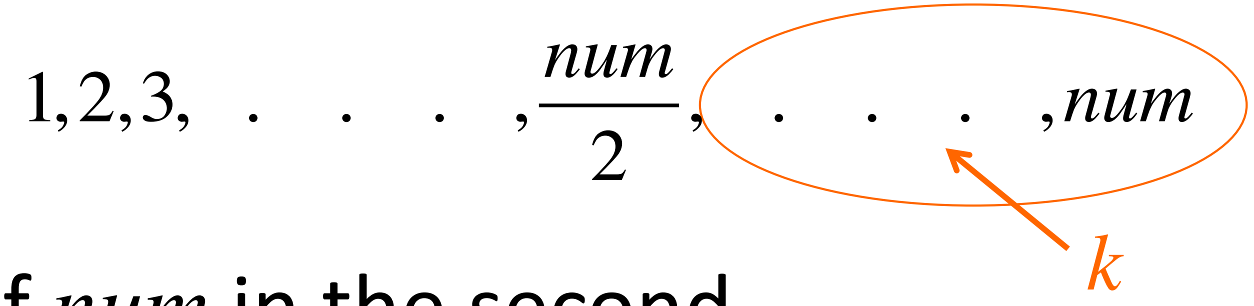

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So: $\frac{num}{k}=1$, therefore $k = num$.

Primality Testing

$$1,2,3,\ .\ \ .\ \ .\ ,\frac{num}{2},\underbrace{.\ \ .\ \ .\ ,num}_{k}$$

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Therefore $d<2$. Since d is a divisor, we get that $d=1$.

So: $\frac{num}{k}=1$, therefore $k = num$.

This shows that the only divisor in the second half of the range is num itself.

Primality Testing

Version I: $\underbrace{1,2,3,\ .\ \ .\ \ .\ ,num}$

Version II: $\underbrace{1,2,3,\ .\ \ .\ \ .\ ,\frac{num}{2}},\ .\ \ .\ \ .\ ,num$

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Version III:

Primality Testing

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num = 100: 1 2 4 5 10 20 25 50 100

Primality Testing

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num = 100: 1 2 4 5 10 20 25 $\underset{\nwarrow \frac{100}{2}}{50}$ 100

Primality Testing

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$$num = 100:\quad 1\quad 2\quad 4\quad 5\quad \underset{\substack{\nwarrow\\ \sqrt{100}}}{10}\quad 20\quad 25\quad \underset{\substack{\nwarrow\\ \frac{100}{2}}}{50}\quad 100$$

Primality Testing

1 2 4 5 10 20 25 50 100

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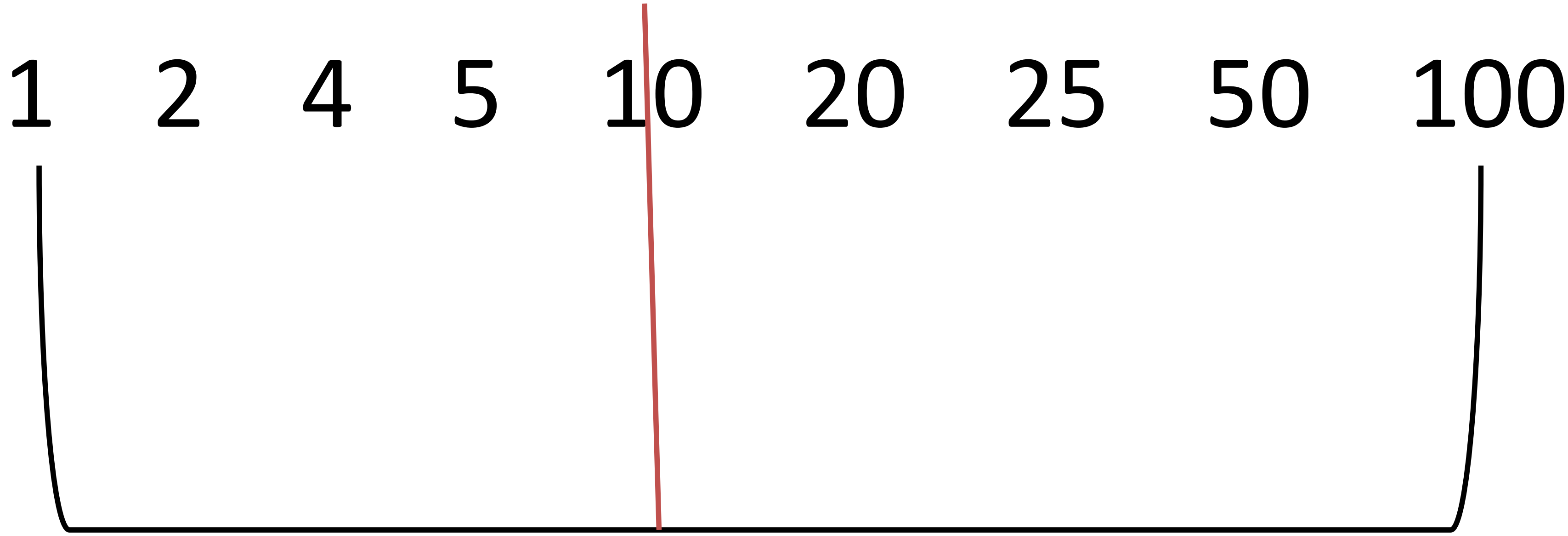
Primality Testing



1 2 4 5 10 20 25 50 100

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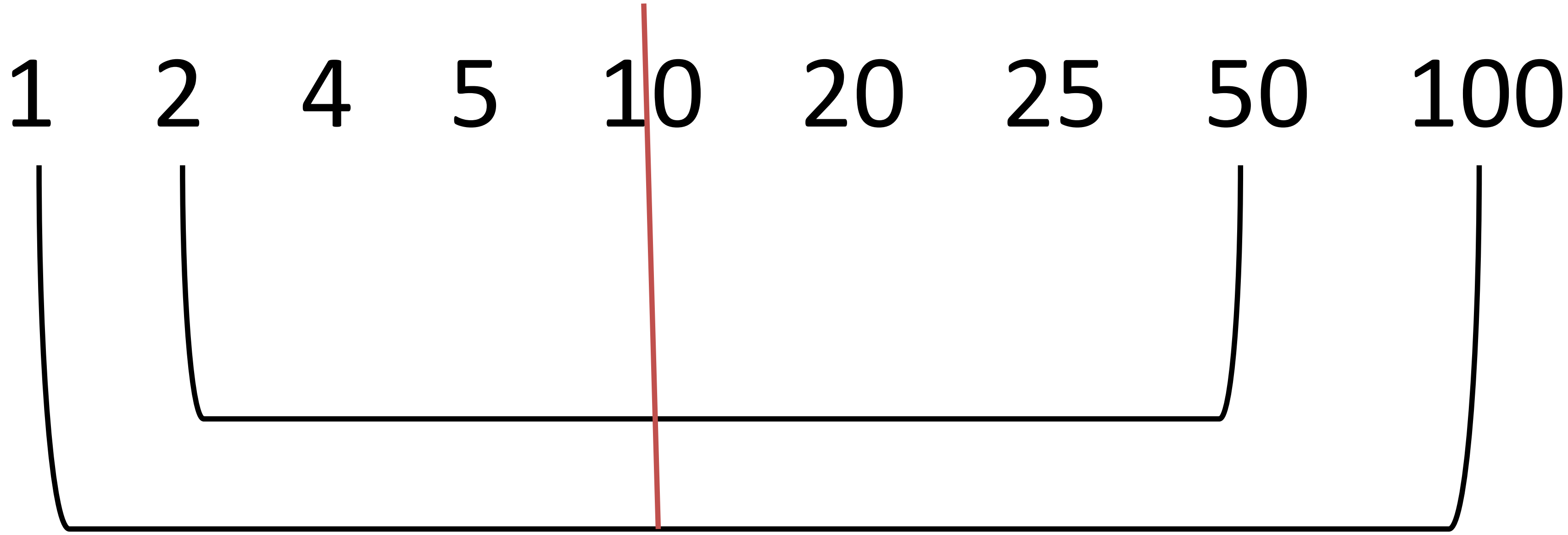

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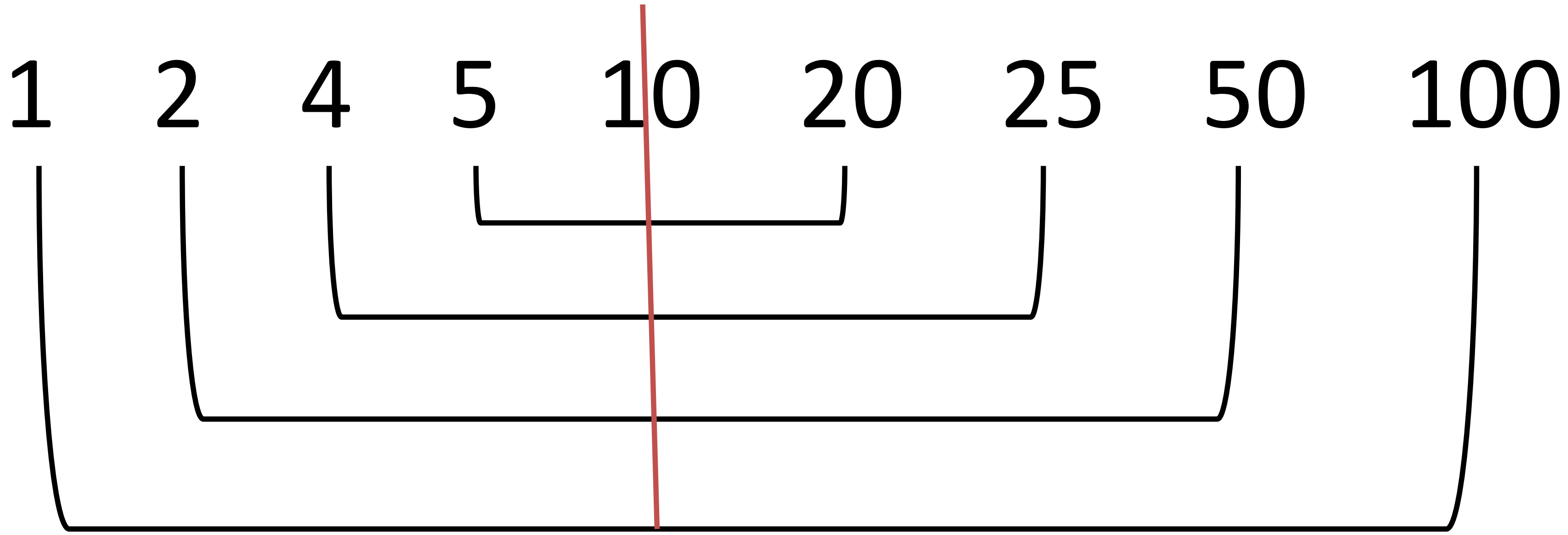

1 2 4 5 10 20 25 50 100

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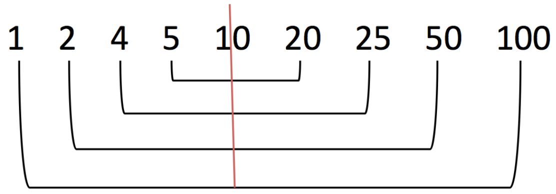

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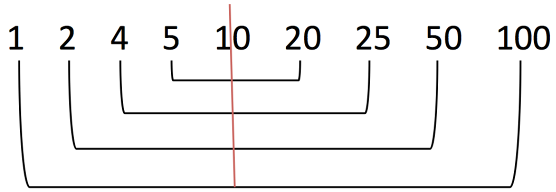

$$1,2,3,\ldots,\sqrt{num}, \qquad . \qquad . \qquad . \qquad ,num$$

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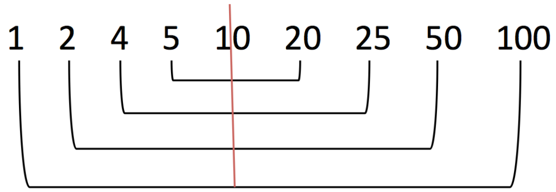

$$1,2,3,\ldots,\sqrt{num},\quad .\quad .\quad .\quad ,num$$

$k,\ d$

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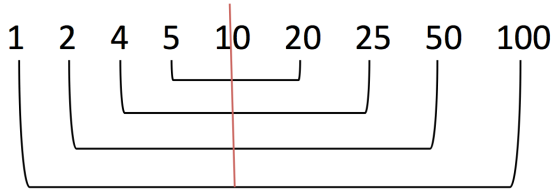

$$1,2,3,\ldots,\sqrt{num}, \quad . \quad . \quad . \quad ,num$$

k, d

Let k and d be complementary divisors of num

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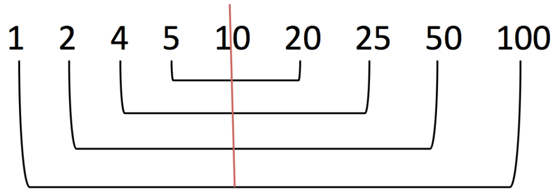

$$1,2,3,\ldots,\sqrt{num},\quad .\quad .\quad .\quad ,num$$

k, d

Let k and d be complementary divisors of num, and assume that they are both greater than $\sqrt{num}$

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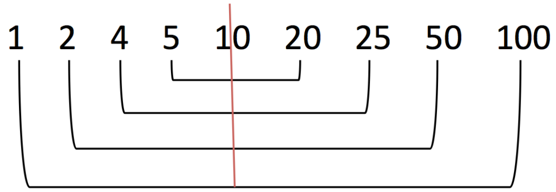

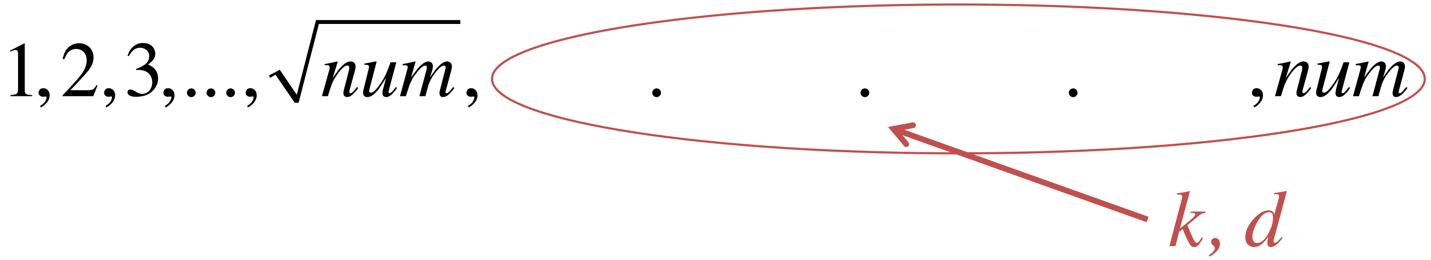

$$1,2,3,\ldots,\sqrt{num},\quad .\quad\quad .\quad\quad .\quad\quad ,num$$

k, d

Let k and d be complementary divisors of num, and assume that they are both greater than $\sqrt{num}$

We therefore have:

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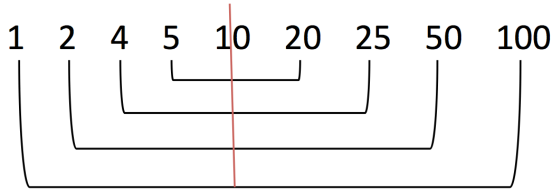

$$1,2,3,\ldots,\sqrt{num},\quad .\quad .\quad .\quad ,num$$

k, d

Let k and d be complementary divisors of num, and assume that they are both greater than $\sqrt{num}$

We therefore have: $num = k \cdot d$

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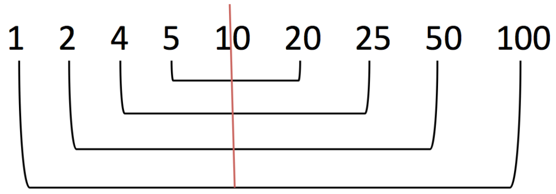

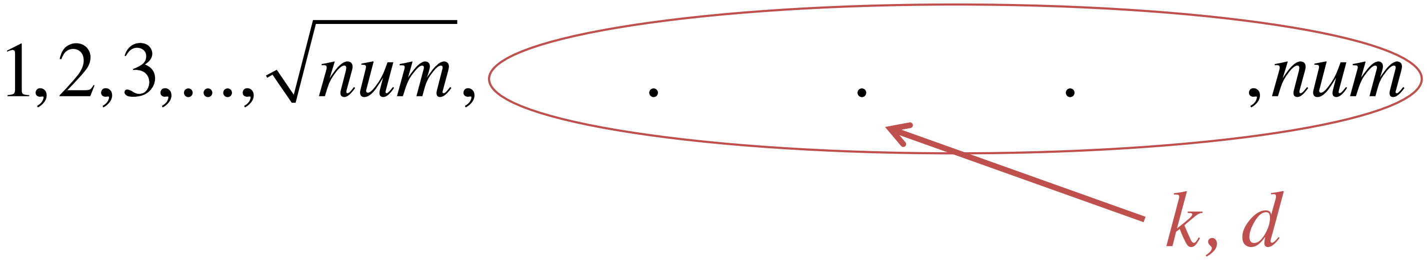

$$1, 2, 3, \ldots, \sqrt{num}, \quad . \quad . \quad . \quad , num$$

k, d

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We therefore have: $num = k \cdot d > \sqrt{num} \cdot \sqrt{num}$

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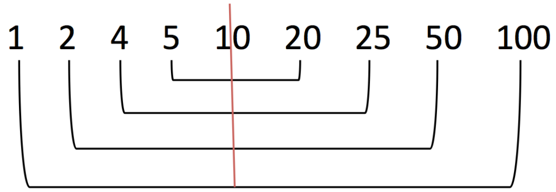
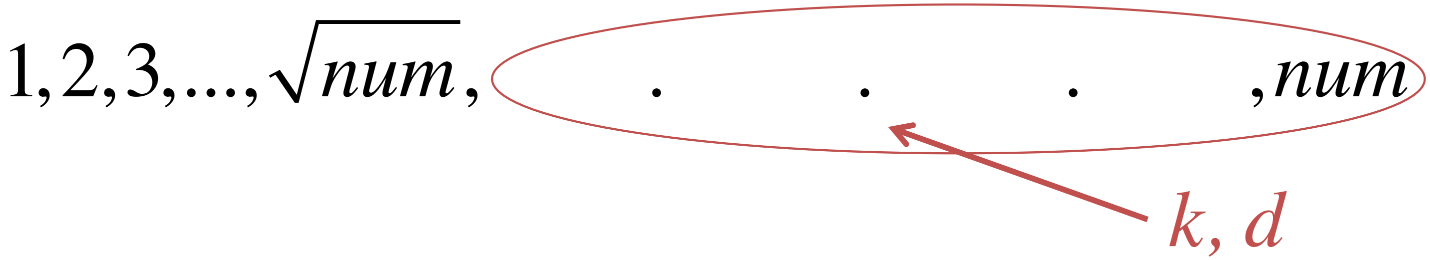




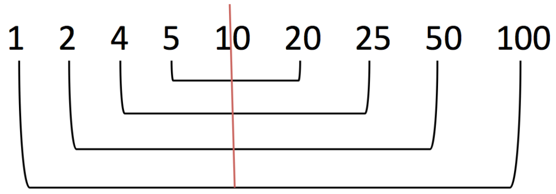
$1,2,3,\ldots,\sqrt{num}, \quad . \quad . \quad . \quad ,num$

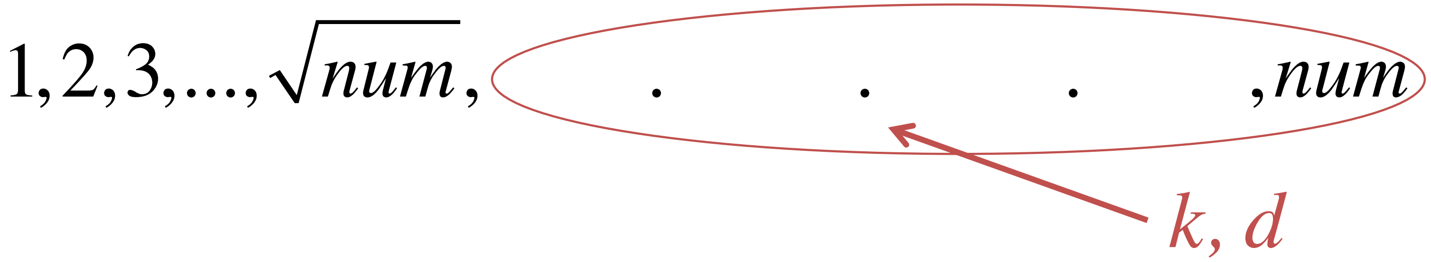
k, d

Let k and d be complementary divisors of num, and assume that they are both greater than $\sqrt{num}$

We therefore have: $num = k \cdot d > \sqrt{num} \cdot \sqrt{num} = num$

Primality Testing


1 2 4 5 10 20 25 50 100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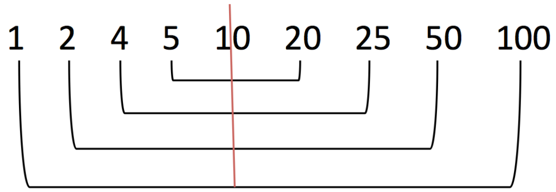

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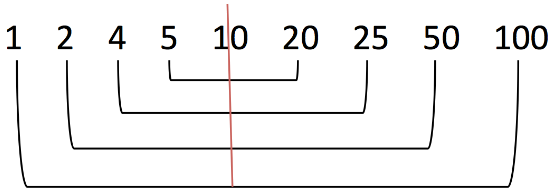

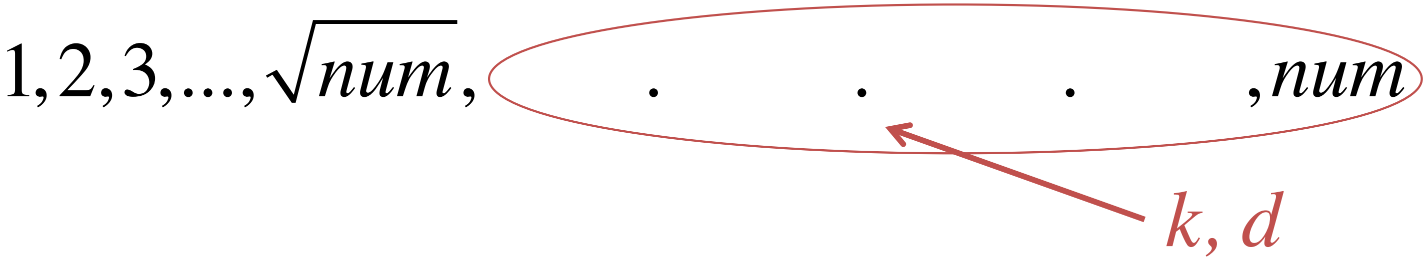

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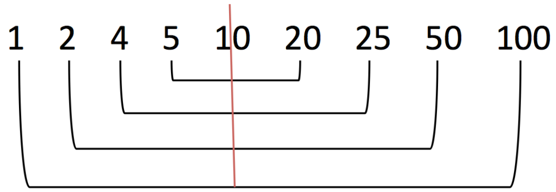

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This shows that at least one in each pair of complementary divisors is less than or equal to $\sqrt{num}$

Primality Testing

Version I: $\underbrace{1,2,3,\quad .\quad .\quad .\quad ,num}$

Version II: $\underbrace{1,2,3,\quad .\quad .\quad .\quad ,\frac{num}{2}}, \quad .\quad .\quad .\quad ,num$

Version III: $\underbrace{1,2,3,\ldots,\sqrt{num}}, \quad .\quad .\quad .\quad ,num$

Runtime Analysis

Runtime Analysis

- The running time depends on the size of the input

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    We count each primitive operation as 1

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- The running time depends on the machine's hardware technology
  - ✓ The abstract model we use, divides the algorithms to classes based on their ”quality”.
    We make asymptotic analysis: look at the order of growth of T(n)

Runtime Analysis

Informal Criteria

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We compare the asymptotic order of the number of primitive operations executed by a process, as a function of its input size

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$$T(n) = 3n^2 + 6n -15$$

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Rule of thumb to get the order of growth:

Runtime Analysis

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Rule of thumb to get the order of growth:

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More Formally . . .

Asymptotic Analysis

O definition

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Definition

Let $f(n)$ and $g(n)$ be two functions mapping positive integers to positive real numbers.

Asymptotic Analysis

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Let $f(n)$ and $g(n)$ be two functions mapping positive integers to positive real numbers.

We say that $f(n)=O(g(n))$

Asymptotic Analysis

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Let $f(n)$ and $g(n)$ be two functions mapping positive integers to positive real numbers.

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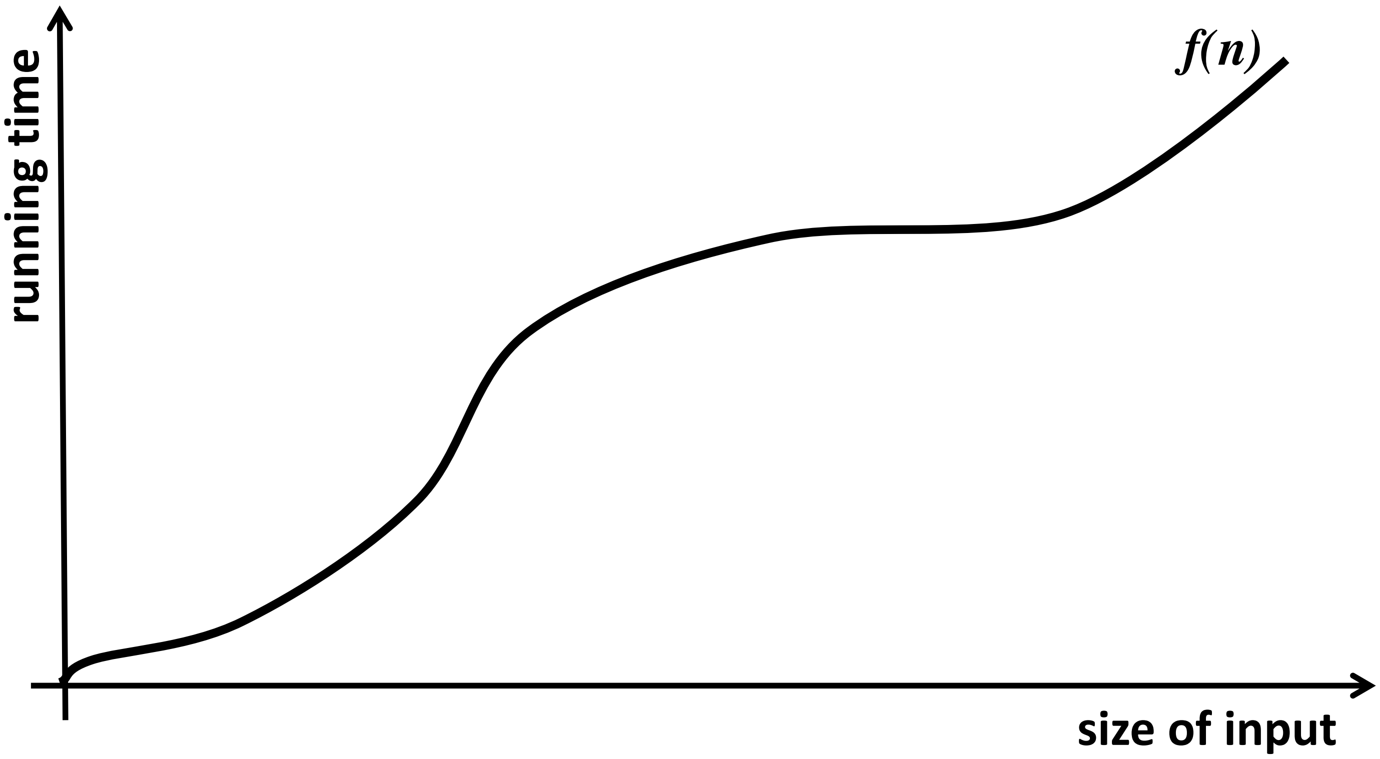

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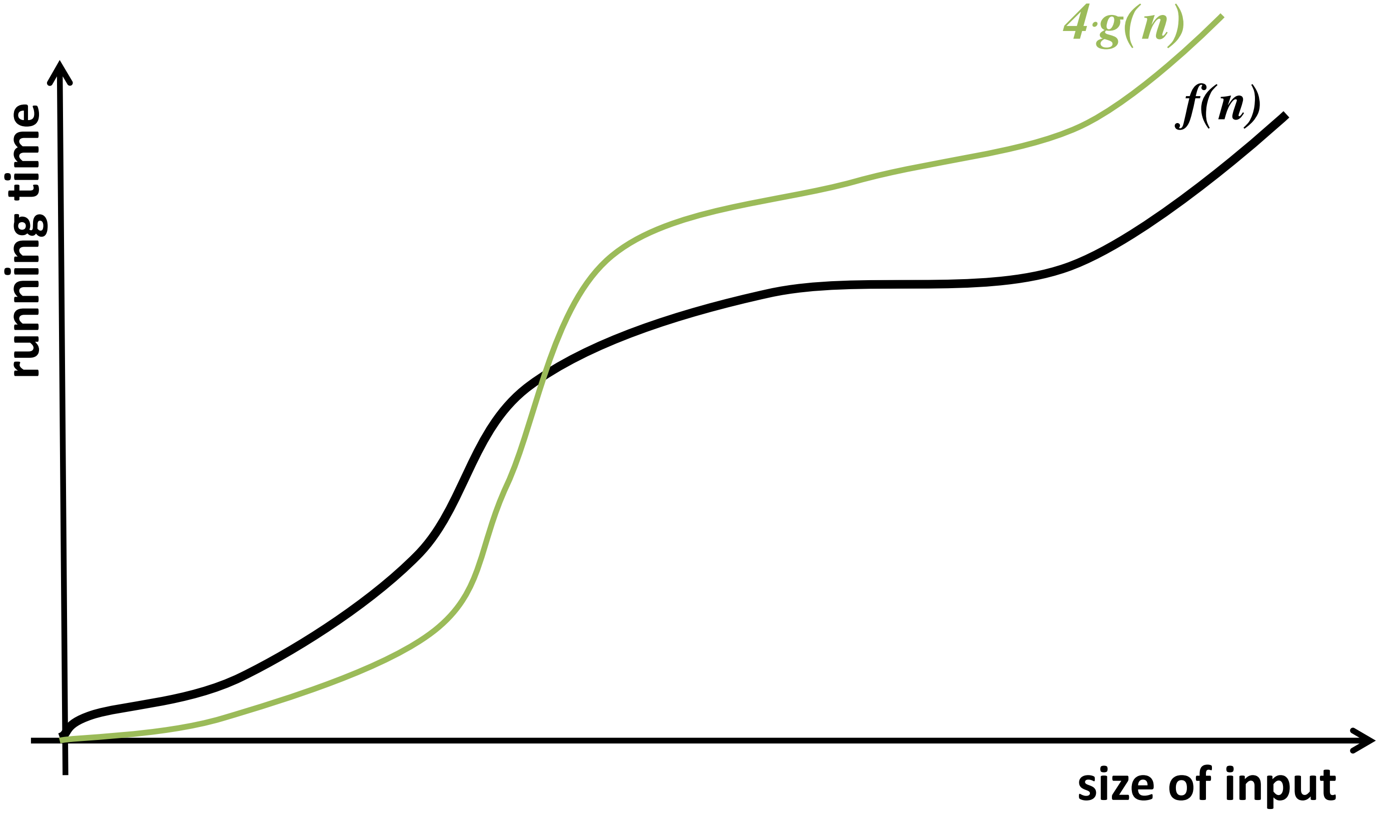

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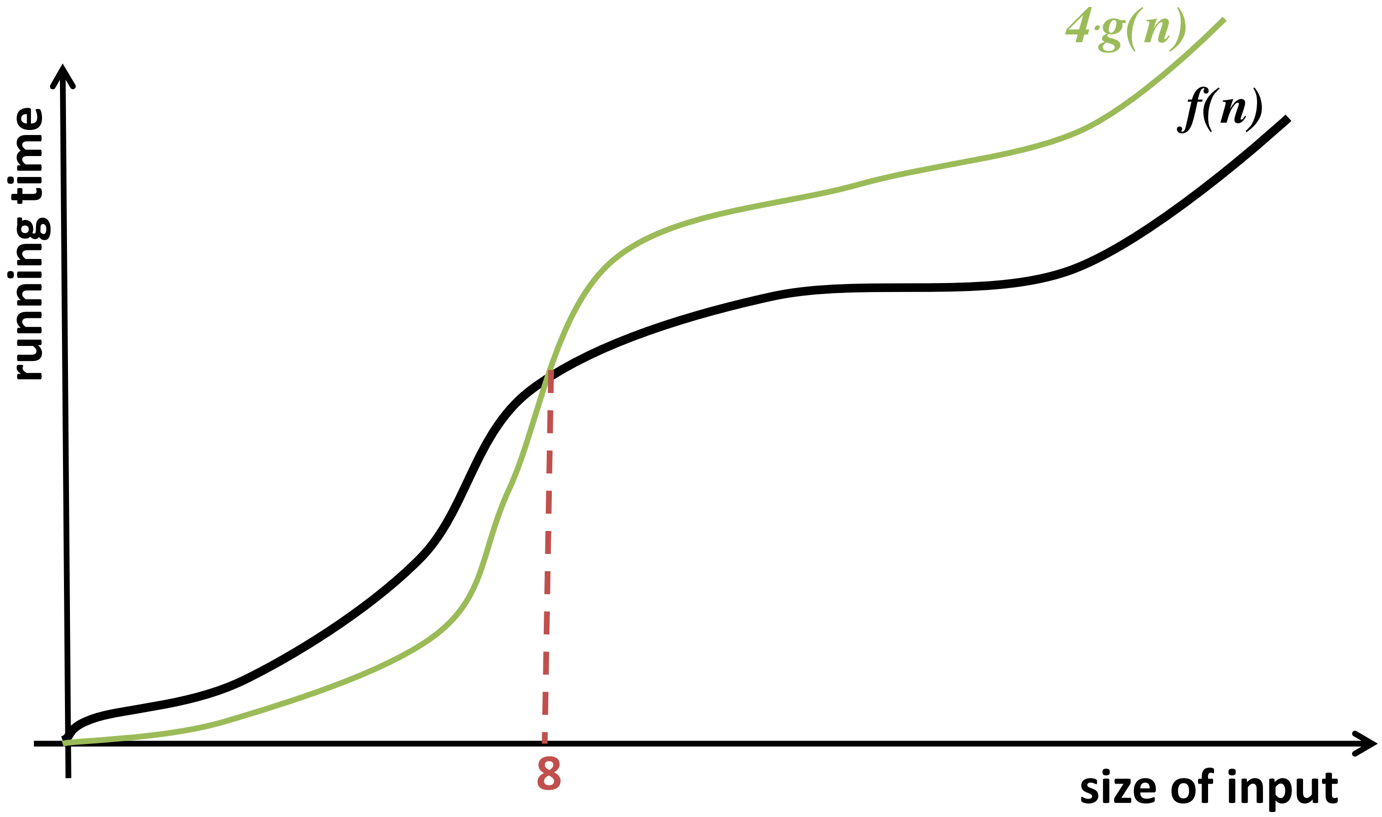

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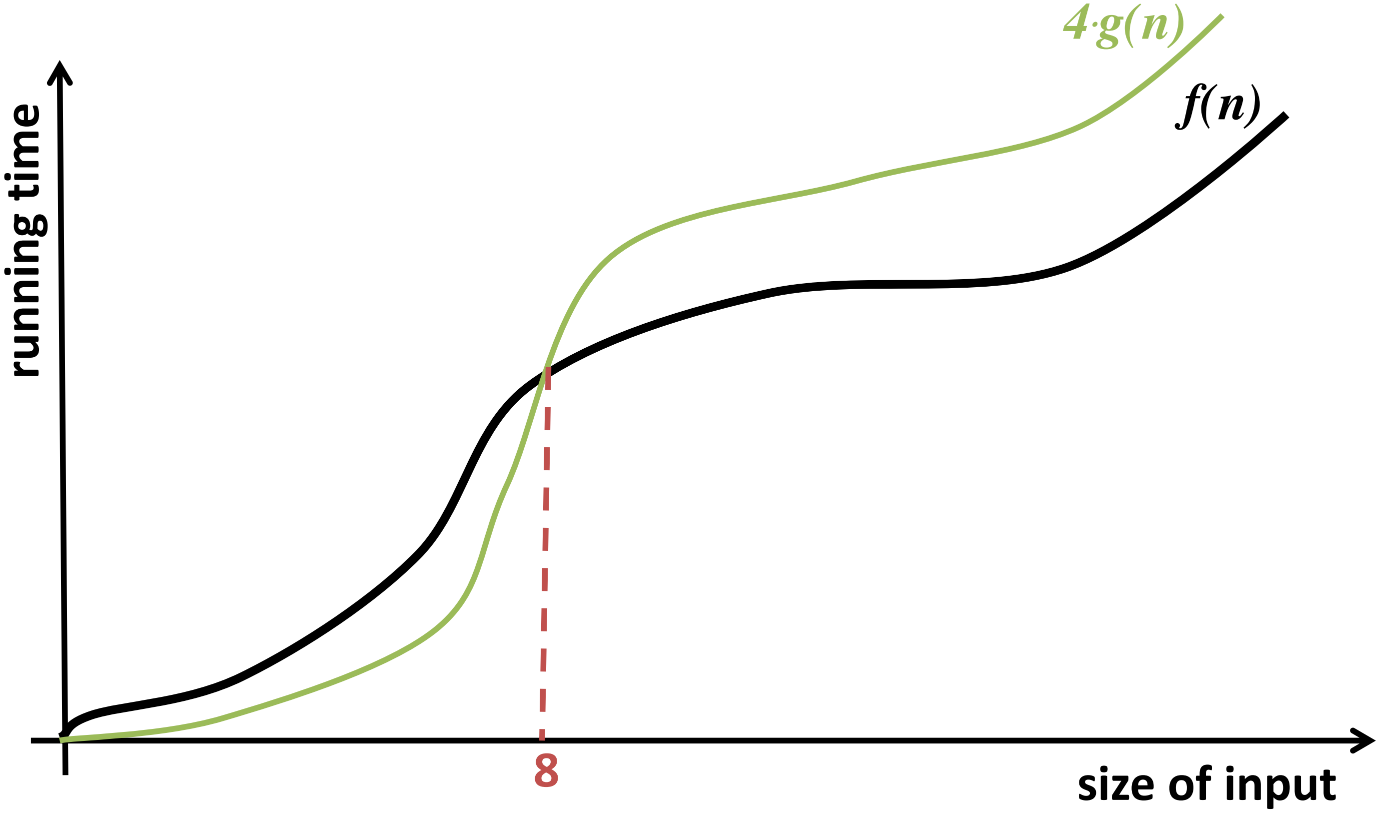
$n_0 = 8$

$c \;\; = 4$



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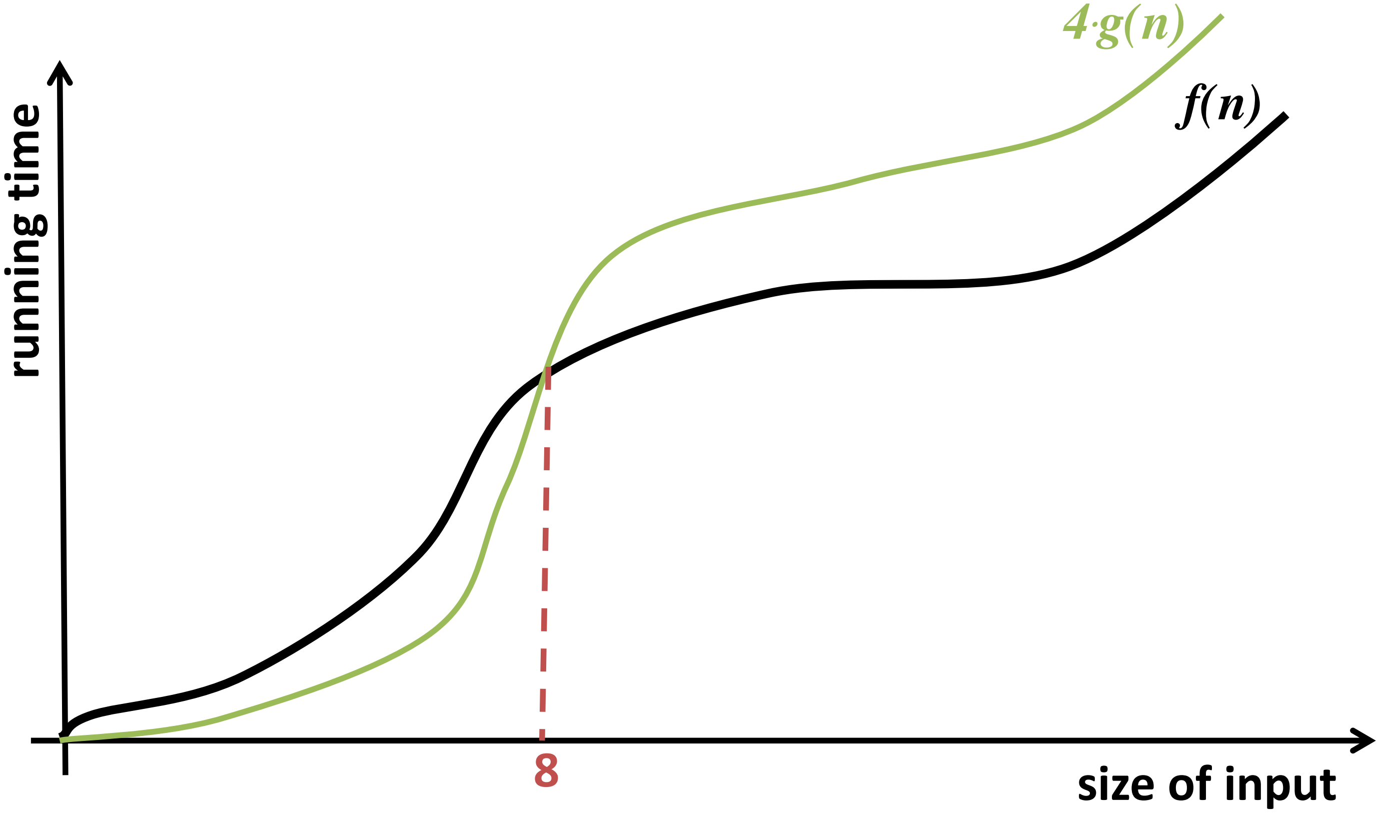
$\Downarrow$

$f(n) = O(g(n))$



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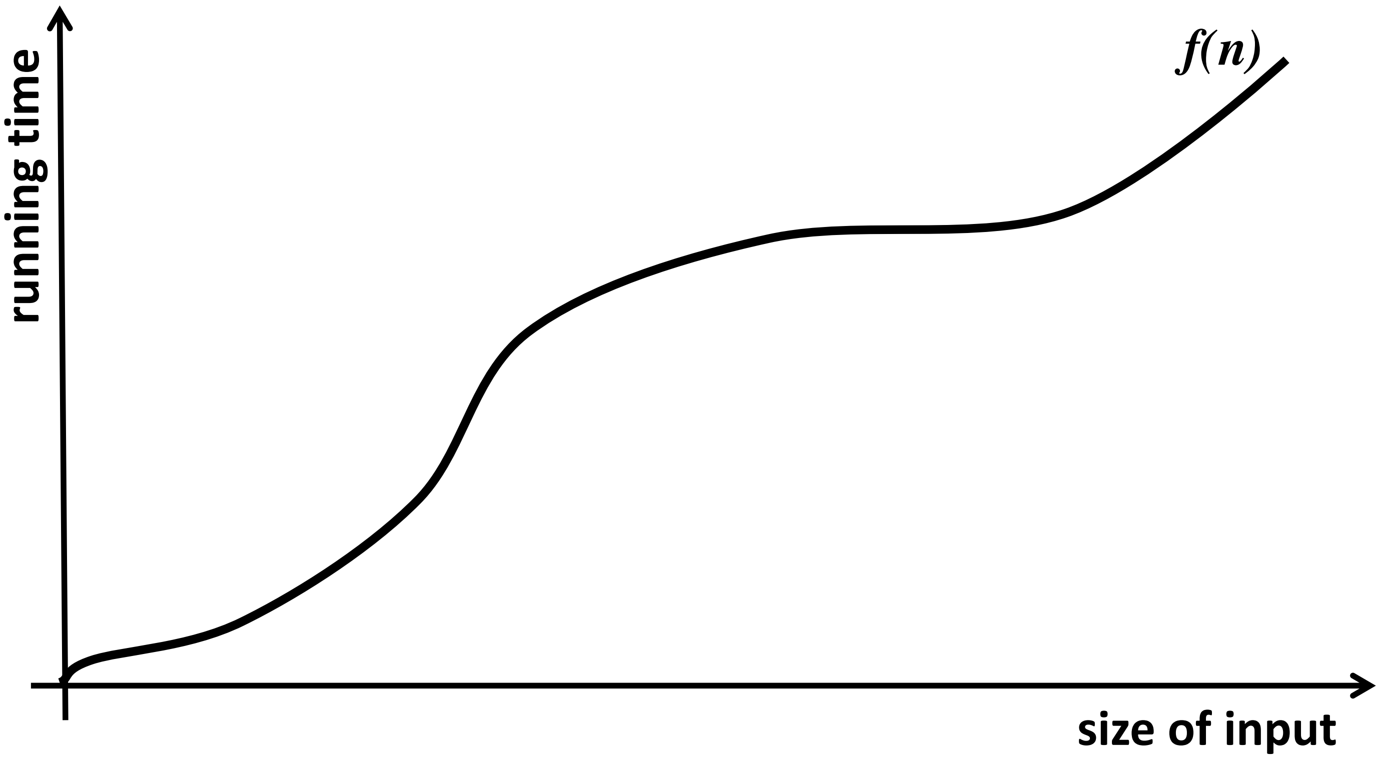

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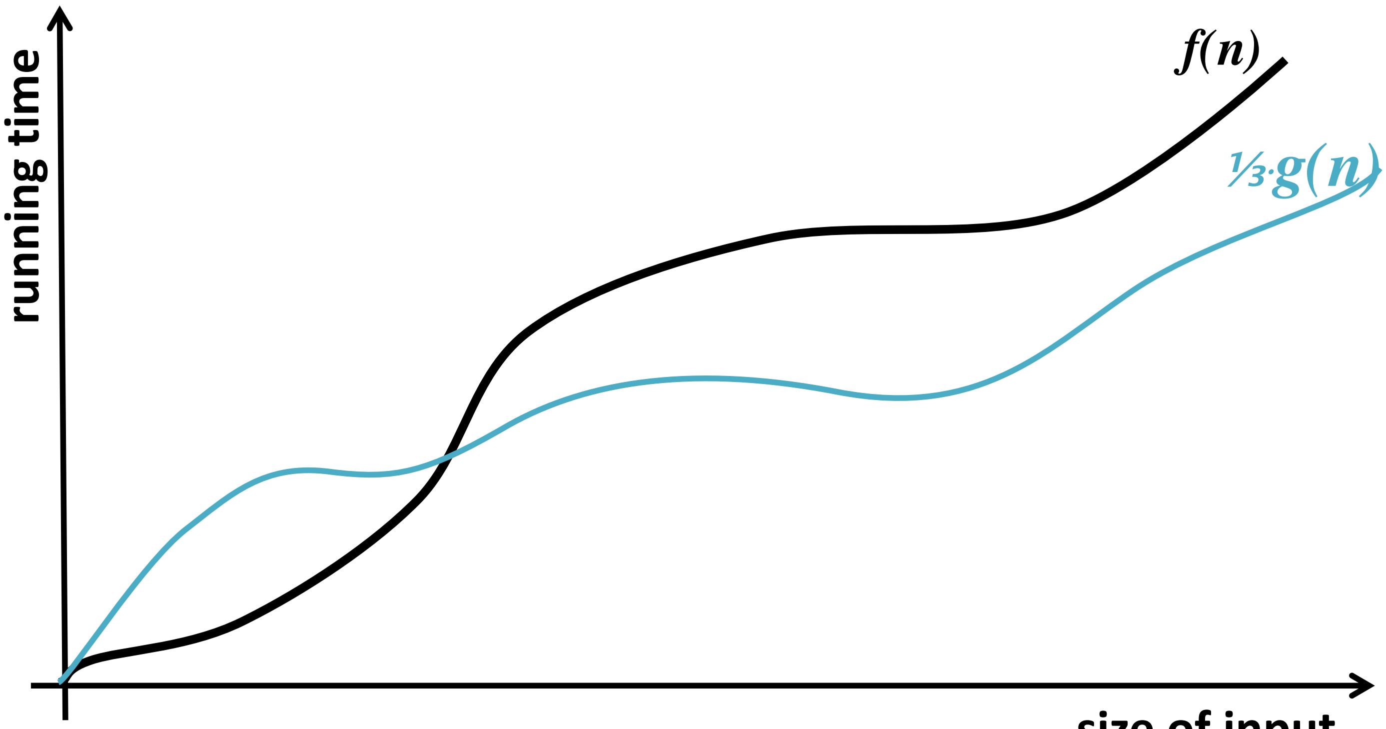

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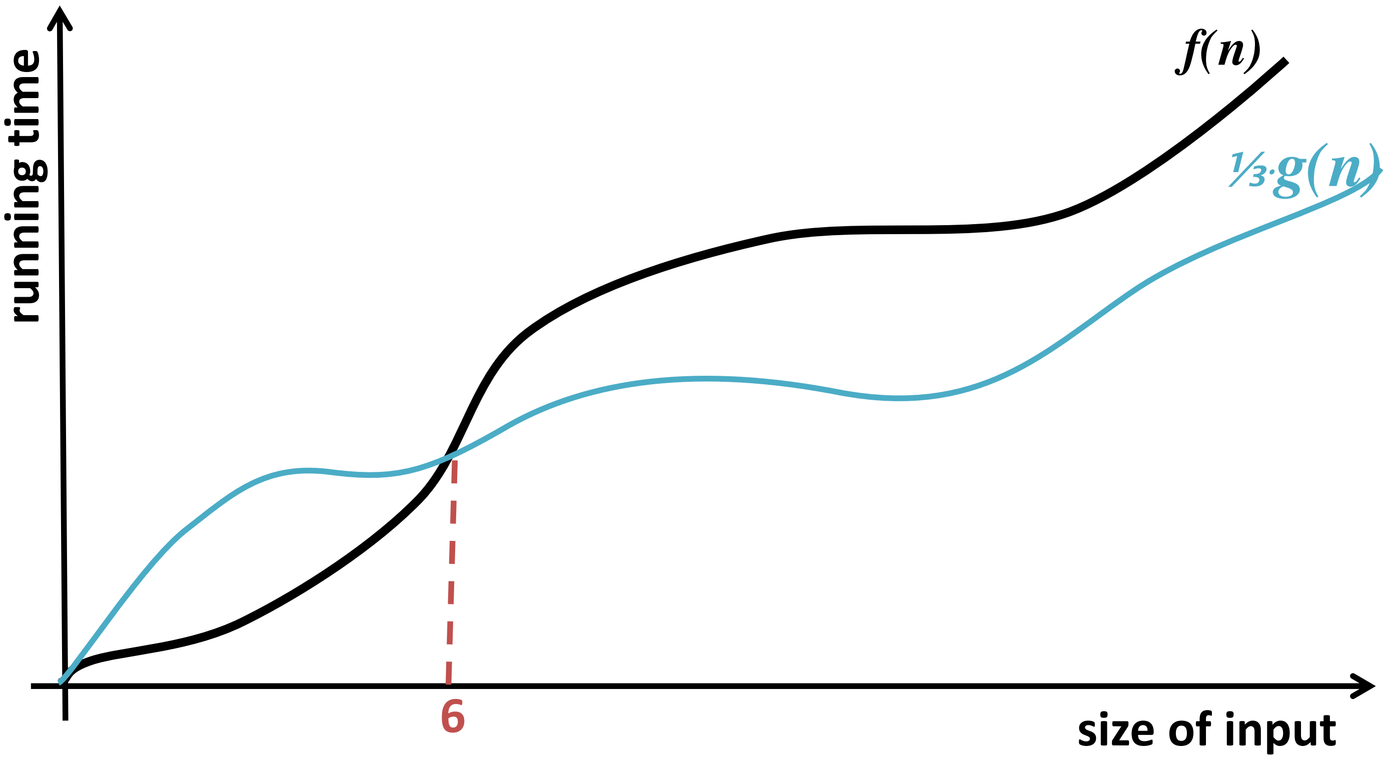

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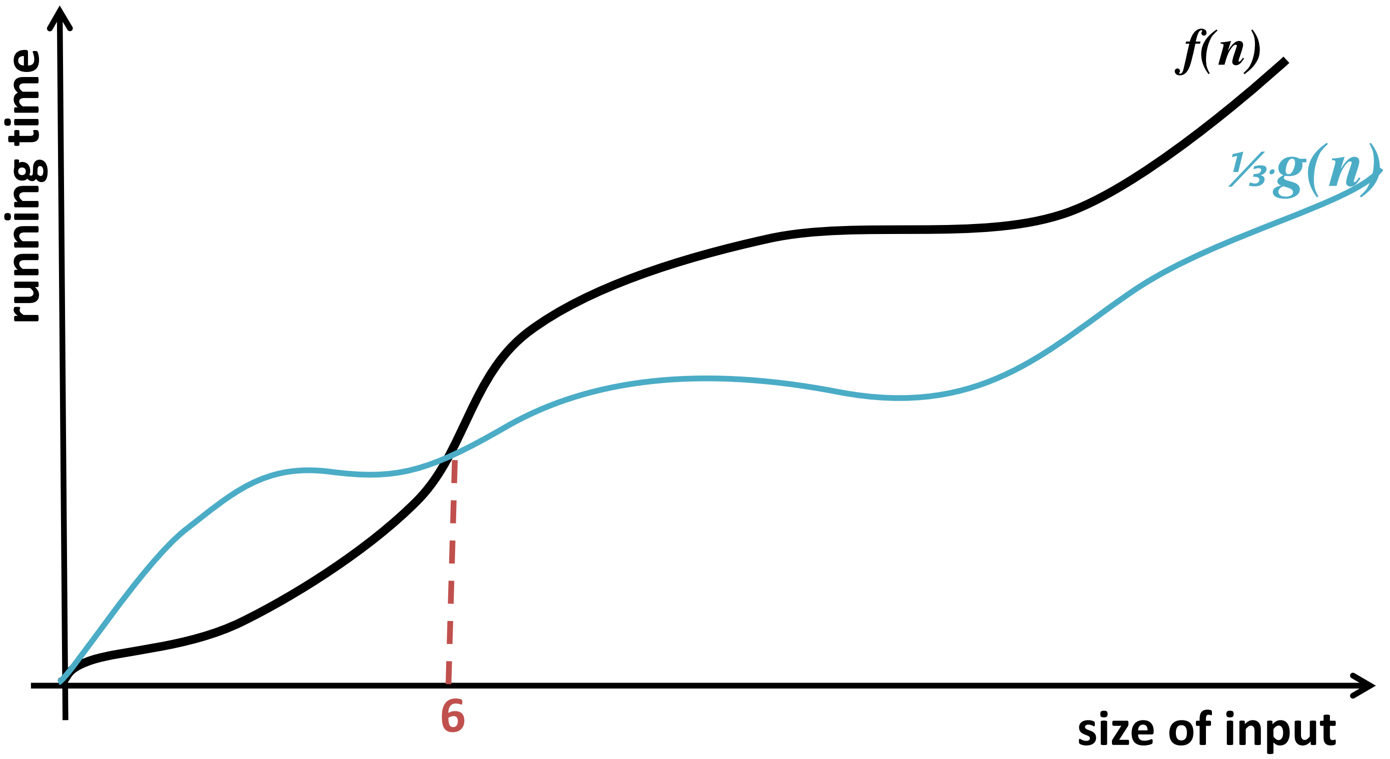
$c_2 = ⅓$



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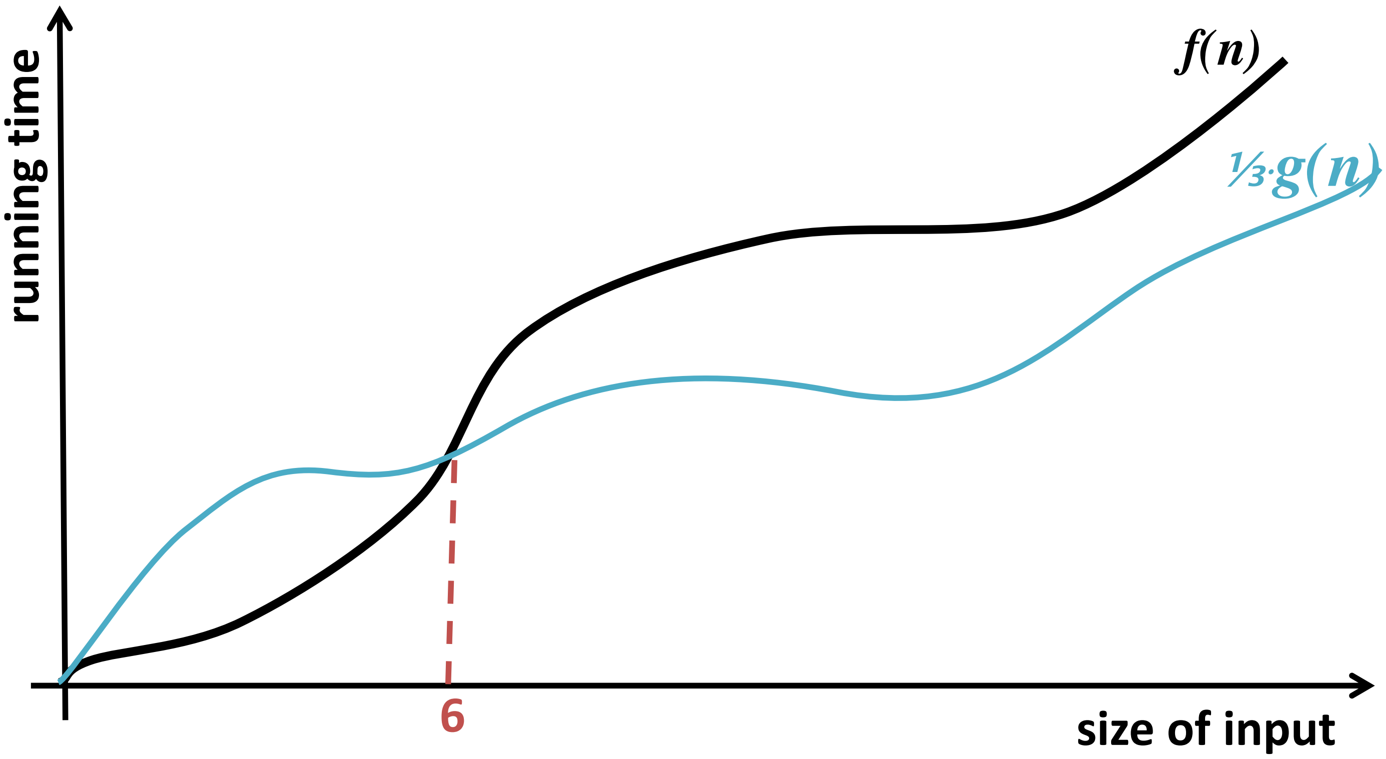
$f(n) = \Omega(g(n))$



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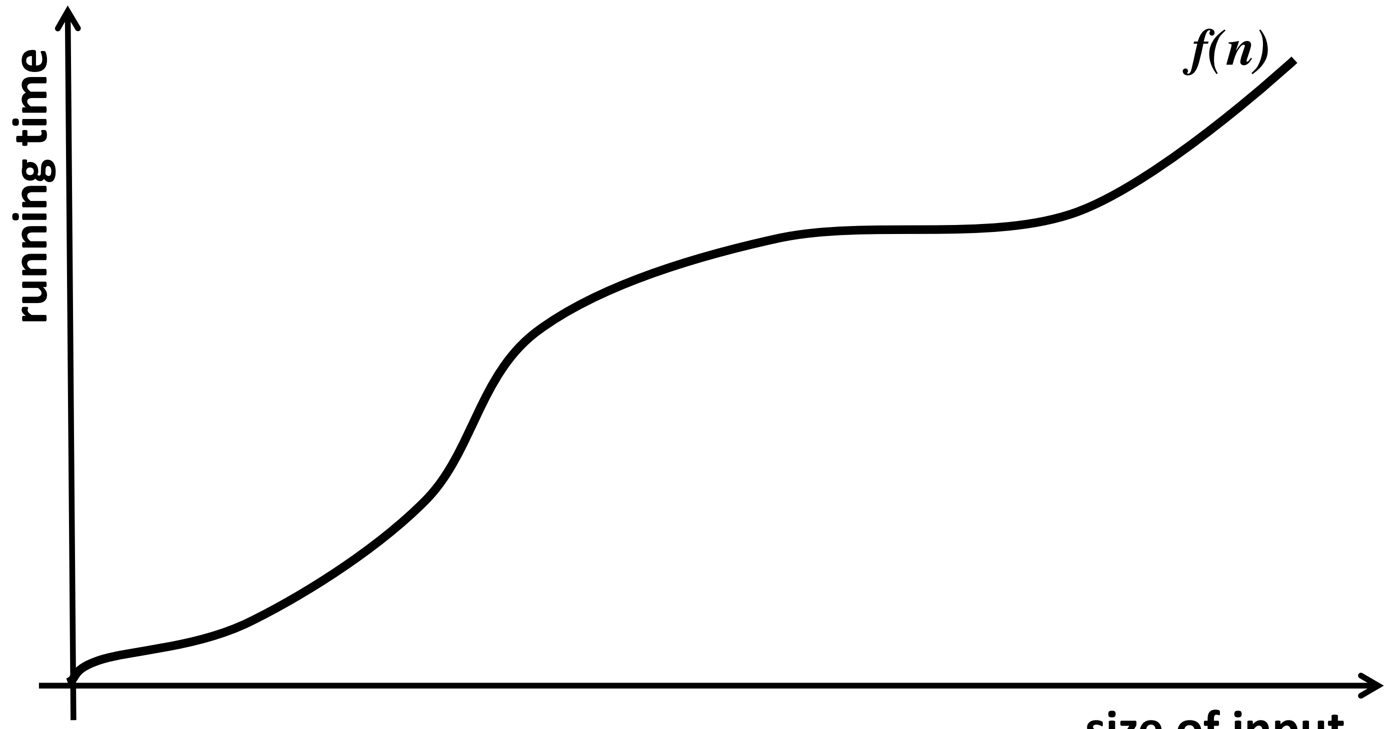

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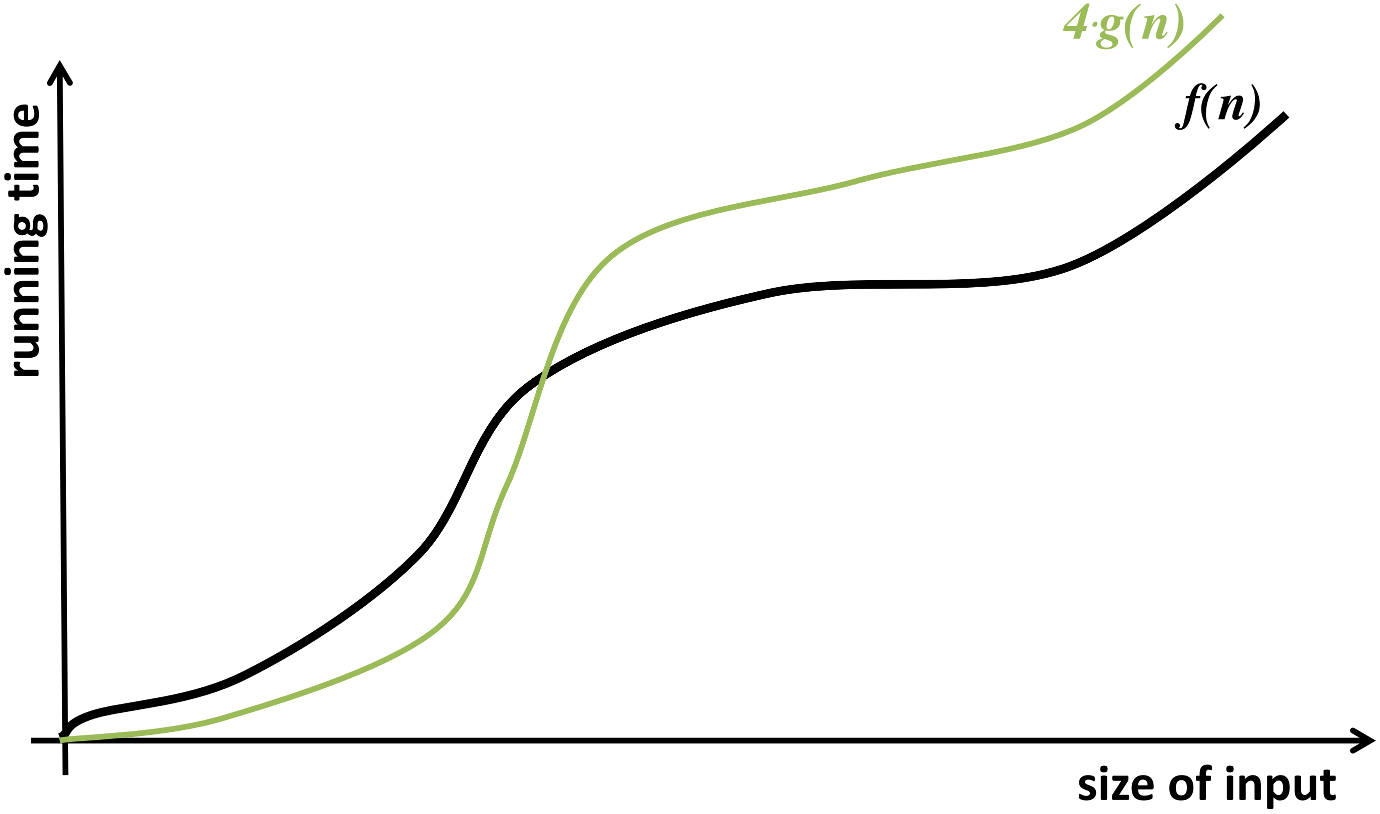

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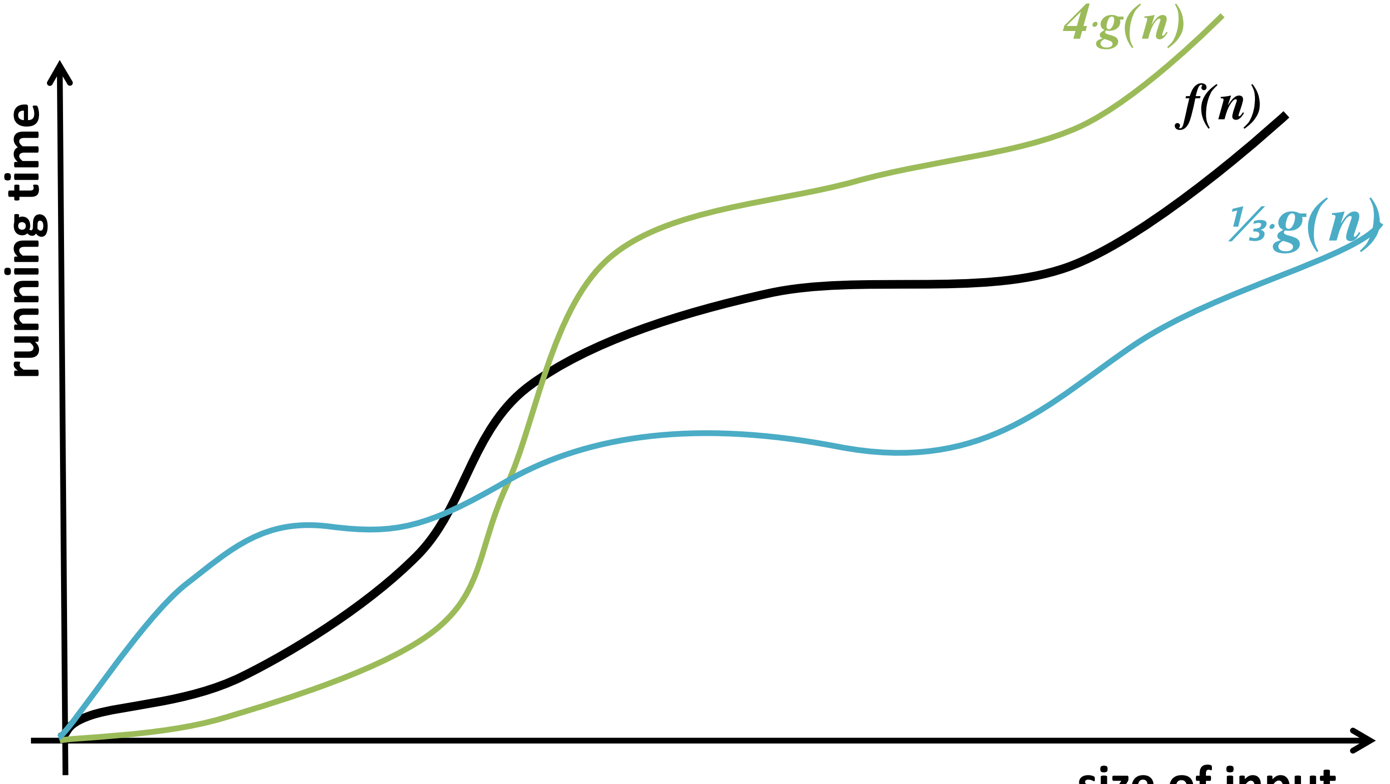

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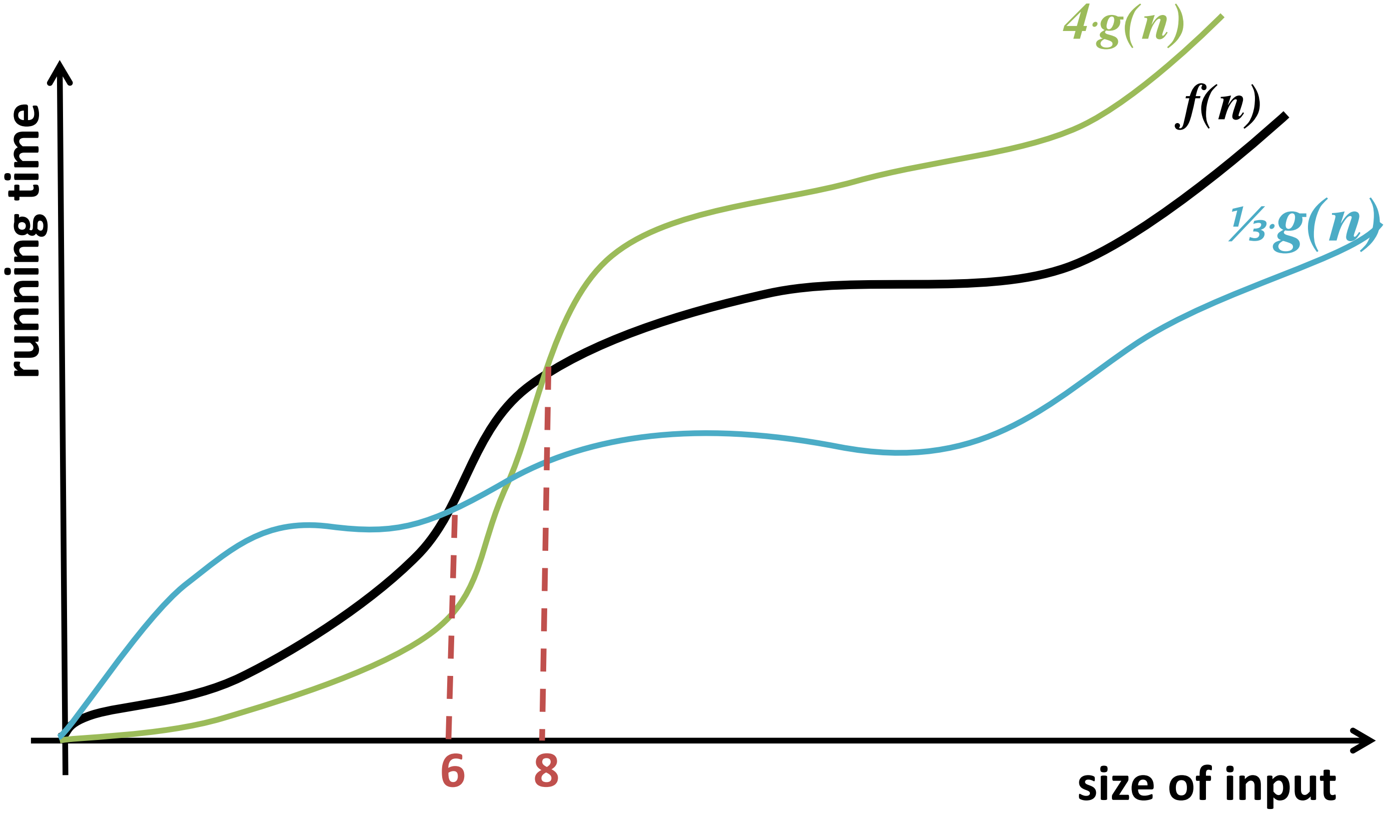

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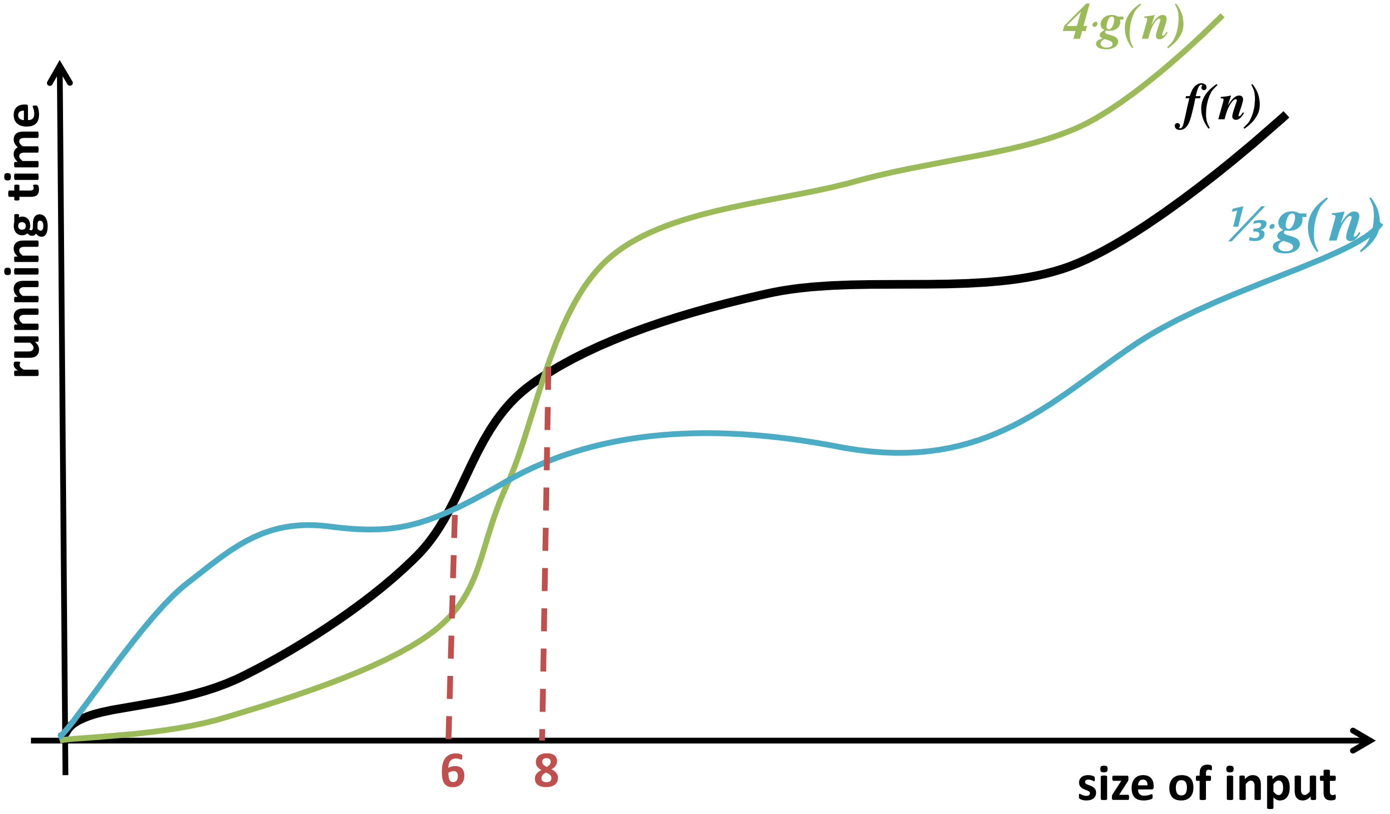
$n_0 = 8$

$c_1 = 4$

$c_2 = ⅓$



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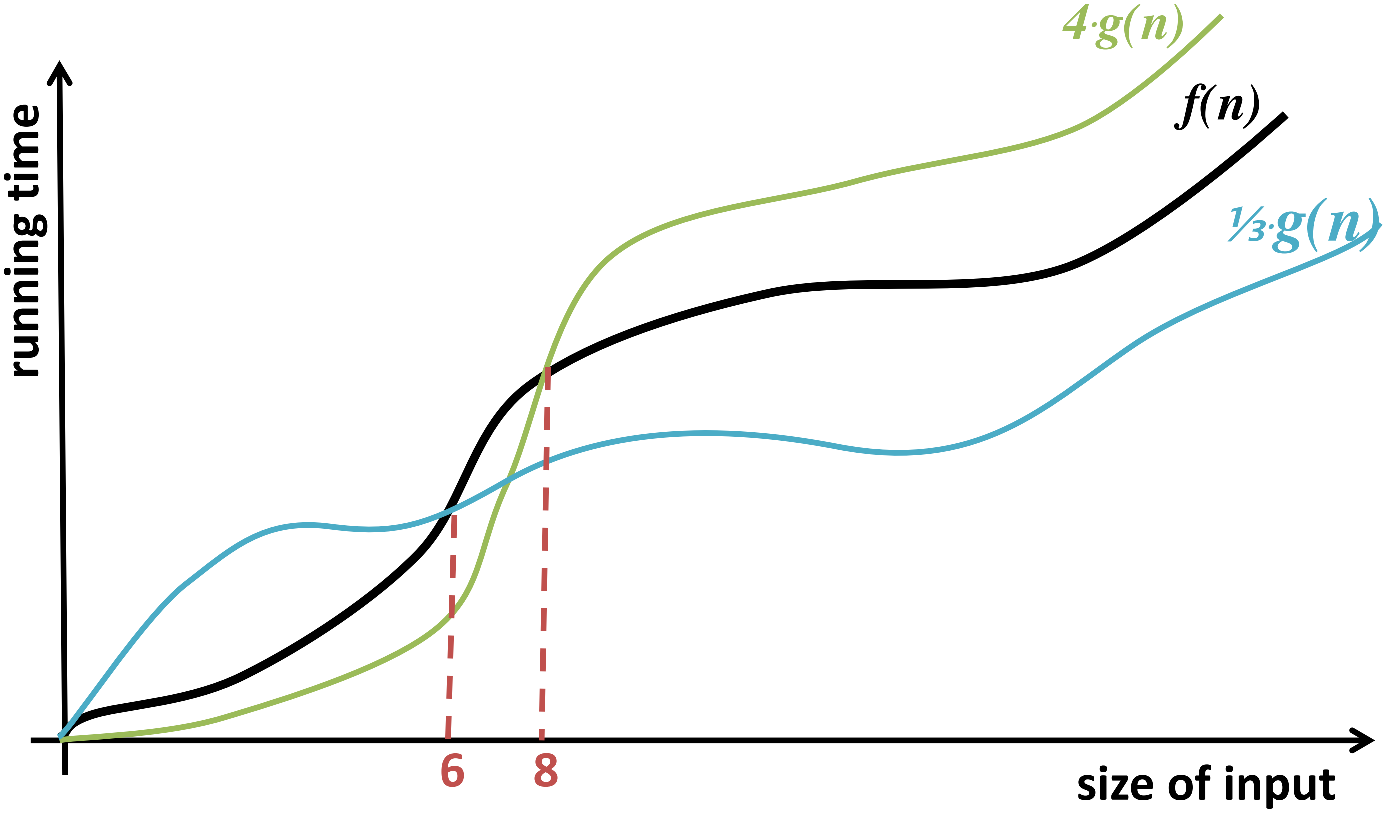
⇩

$f(n) = \theta(g(n))$



Asymptotic Analysis

Θ definition

Show that: $3n^2 + 6n - 15 = \theta(n^2)$

Asymptotic Analysis

Θ definition

Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

Asymptotic Analysis

Θ definition

Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

Proof:

Asymptotic Analysis

Θ definition

Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

Proof:

if we take c_1 = _____

c_2 = _____

n_0 = _____

Asymptotic Analysis

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Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

Proof:

if we take c_1 = _____

c_2 = _____

n_0 = _____

Then for all $n \geq n_0$ we have:

Asymptotic Analysis

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Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

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$$3n^2 + 6n - 15 \leq 3n^2 + 6n$$

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Then for all $n \geq n_0$ we have:

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Asymptotic Analysis

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Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

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if we take $c_1 =$ _____

$c_2 =$ _____

$n_0 =$ _____

Then for all $n \geq n_0$ we have:

$$3n^2 + 6n - 15 \leq 3n^2 + 6n \leq 3n^2 + 6n^2 = 9n^2$$

Asymptotic Analysis

Θ definition

<u>Show that:</u> $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

<u>Proof:</u>

if we take $c_1 = \underline{\ 9\ \ \ \ }$

$c_2 = \underline{\ \ \ \ \ \ \ }$

$n_0 = \underline{\ \ \ \ \ \ \ }$

Then for all $n \geq n_0$ we have:

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Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

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if we take c_1 = $\underline{\ 9\ }$

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Then for all $n \geq n_0$ we have:

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$c_2 = \underline{\ \ \ \ \ \ \ \ }$

$n_0 = \underline{\ \ \ \ \ \ \ \ }$

Then for all $n \geq n_0$ we have:

$$3n^2 \leq 3n^2 + 6n - 15 \leq 3n^2 + 6n \leq 3n^2 + 6n^2 = 9n^2$$

$6n - 15 \geq 0$

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Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

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n_0 = _____

Then for all $n \geq n_0$ we have:

$$3n^2 \leq 3n^2 + 6n - 15 \leq 3n^2 + 6n \leq 3n^2 + 6n^2 = 9n^2$$

$6n - 15 \geq 0$

$\updownarrow$

$6n \geq 15$

Asymptotic Analysis

Θ definition

Show that: $\underbrace{3n^2 + 6n -15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

Proof:

if we take $c_1 = \underline{\quad 9 \quad}$

$c_2 =$ _____

$n_0 =$ _____

Then for all $n \geq n_0$ we have:

$$3n^2 \leq 3n^2 + 6n - 15 \leq 3n^2 + 6n \leq 3n^2 + 6n^2 = 9n^2$$

$$\begin{gathered} 6n - 15 \geq 0 \\ \Updownarrow \\ 6n \geq 15 \\ \Updownarrow \\ n \geq 2.5 \end{gathered}$$

Asymptotic Analysis

Θ definition

__Show that:__ $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

__Proof:__

if we take $c_1 = \underline{\ 9\ \ \ \ }$

$c_2 = \underline{\ \ \ \ \ \ \ }$

$n_0 = \underline{\ 3\ \ \ \ }$

Then for all $n \geq n_0$ we have:

$$3n^2 \leq 3n^2 + 6n - 15 \leq 3n^2 + 6n \leq 3n^2 + 6n^2 = 9n^2$$

$$6n - 15 \geq 0 \Updownarrow 6n \geq 15 \Updownarrow n \geq 2.5$$

Asymptotic Analysis

Θ definition

Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

Proof:

if we take $c_1 = \underline{\;9\;\;\;}$

$c_2 = \underline{\;3\;\;\;}$

$n_0 = \underline{\;3\;\;\;}$

Then for all $n \geq n_0$ we have:

$$3n^2 \leq 3n^2 + 6n - 15 \leq 3n^2 + 6n \leq 3n^2 + 6n^2 = 9n^2$$

$$6n - 15 \geq 0$$
$$\Updownarrow$$
$$6n \geq 15$$
$$\Updownarrow$$
$$n \geq 2.5$$

Asymptotic Analysis

Θ definition

Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

Proof:

if we take $c_1 =$ 9

$c_2 =$ 3

$n_0 =$ 3

Then for all $n \geq n_0$ we have:

$$3n^2 \leq 3n^2 + 6n - 15 \leq 3n^2 + 6n \leq 3n^2 + 6n^2 = 9n^2$$

$$6n - 15 \geq 0 \Updownarrow 6n \geq 15 \Updownarrow n \geq 2.5$$

$$\Downarrow$$

$$3n^2 \leq 3n^2 + 6n - 15 \leq 9n^2$$

Asymptotic Analysis

Θ definition

Show that: $\underbrace{3n^2 + 6n - 15}_{f(n)} = \theta(\underbrace{n^2}_{g(n)})$

Proof:

if we take

c_1 = 9

c_2 = 3

n_0 = 3

Then for all $n \geq n_0$ we have:

$$3n^2 \leq 3n^2 + 6n - 15 \leq 3n^2 + 6n \leq 3n^2 + 6n^2 = 9n^2$$

$$6n - 15 \geq 0 \Updownarrow 6n \geq 15 \Updownarrow n \geq 2.5$$

$$\Downarrow$$

$$3n^2 \leq 3n^2 + 6n - 15 \leq 9n^2$$

Therefore: $3n^2 + 6n - 15 = \theta(n^2)$